Reply to `Comment on ``Large-*N* theory of strongly commensurate dirty-bosons: absence of a transition in two dimensions'' '

2002 J. Phys. A: Math. Gen. 35 2523

(http://iopscience.iop.org/0305-4470/35/10/402)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 2523–2525 PII: S0305-4470(02)33011-7

COMMENT

Reply to 'Comment on "Large-N theory of strongly commensurate dirty-bosons: absence of a transition in two dimensions" '

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Received 23 January 2002
Published 1 March 2002
Online at stacks.iop.org/JPhysA/35/2523

Abstract
We show that the self-consistency requirement on random potentials causes the decrease of correlations with disorder in the large-N theory of commensurate dirty bosons, leaving the possibility that there is no superfluid transition intact.

PACS numbers: 05.30.Jp, 11.10.−z, 64.60.−i, 72.80.Ng, 73.20.Jc, 73.20.Fz

In a recent paper [1], we argued in favor of the absence of a phase transition in a model of two-dimensional strongly commensurate dirty bosons in the large-N limit. By studying the model perturbatively in weak disorder, we showed first that a Gaussian random potential is imperfectly screened by interactions, and hence the ground state in the effective Anderson localization problem should always remain localized. This, we argued, prohibits the transition into the superfluid state. We further provided evidence from the numerical calculations supportive of our conclusion.

In his comment [2], Hastings argues that non-existence of the transition would violate the correlation inequalities introduced by Griffiths, and thus concludes that there must be the superfluid phase in the model. We argue here that in doing so he neglected the effect of screening of the random potential by the interactions, which is essential in the large-N theory. We then proceed to show that including this effect invalidates Hastings' conclusion. Griffiths inequalities actually imply nothing for the existence of the transition in the model in question.

We considered the action

$$S[\psi] = \int \mathrm{d}^D\vec{x}\,\mathrm{d}\tau \left\{ (\partial_\tau \psi(\vec{x},\tau))^2 + (\nabla\psi(\vec{x},\tau))^2 + (V(\vec{x}) - \mu)\psi^2(\vec{x},\tau) + \frac{\lambda}{N}\psi^4(\vec{x},\tau) \right\} \quad (1)$$

with ψ an N-component field, and $V(\vec{x})$ random, $V(\vec{x}) \in [-\delta, \delta]$, and $\lambda > 0$. To make use of the Griffiths inequalities [3] for general N we first perform a Hubbard–Stratonovich transformation on our action to deal with the quartic term. Correlations of ψ are then obtained from the effective Gaussian action

$$S_{\mathrm{eff}} = \int \mathrm{d}^D\vec{x}\,\mathrm{d}\tau\,\left\{(\partial_\tau\psi(\vec{x},\tau))^2 + (\nabla\psi(\vec{x},\tau))^2 + (V(\vec{x}) - \mu + \chi(\vec{x}))\psi^2(\vec{x},\tau)\right\} \tag{2}$$

where the screening of the random potential is determined self-consistently through the saddle-point conditions [1]

$$\chi(\vec{x}) = \lambda\left\langle\vec{x},\tau\left|\frac{1}{-\partial_\tau^2 - \nabla^2 + V(\vec{x}) + \chi(\vec{x}) - \mu}\right|\vec{x},\tau\right\rangle + c^2\phi_0^2(\vec{x}) \tag{3}$$

$$(-\nabla^2 + V(\vec{x}) + \chi(\vec{x}) - \mu)\phi_\alpha(\vec{x}) = \varepsilon_\alpha\phi_\alpha(\vec{x}) \tag{4}$$

$$\varepsilon_0 c = 0. \tag{5}$$

Rewriting the effective action in the discrete lattice form, we get

$$H = \sum_{i,j,\tau}\tilde{J}_{i,j}\psi_\alpha(i,\tau)\psi_\alpha(j,\tau) \tag{6}$$

with $\tilde{J}_{i,i} = (\mu - \tilde{V}(i))$, and $\tilde{V}(i) = V(i) + \chi(i)$. It is now possible to use the correlation inequalities for general N [4], particularly the result

$$\frac{\partial\left\langle\psi_\alpha(i)\psi_\alpha(j)\right\rangle}{\partial\tilde{J}_{kl}} \geqslant 0. \tag{7}$$

Following Hastings, we consider first the pure system by setting $V(i) = 0$ for all i. In this case, $\chi(i) = \chi_0$, independent of i, and the transition occurs when $\mu = \mu_{\mathrm{c,pure}} = \chi_0$, with χ_0 determined self-consistently. Next, we allow $V(i)$ to become weakly random, but with all $V(i) < 0$. This immediately implies that $\chi(i)$ becomes position dependent as well. The main point is that self-consistency yields $\tilde{V}(i) - \mu > 0$, for all i at all μ, even for $V(i) < 0$. In particular, this holds at $\mu = \mu_{\mathrm{c,pure}}$, so all $J_{i,i}$ with disorder become negative, whereas $J_{i,i} \equiv 0$ at the critical point in the pure case. We see this in our numerical solution, and indeed, little thought shows that this should in general be true. In the limit of infinitely strong disorder (compared to the gradient term in (1)) the self-consistent equations decouple at different sites, and it is trivial to see that $\tilde{V}(i) - \mu > 0$ in this local limit. For very weak disorder, on the other hand, if $\tilde{V}(m) - \mu < 0$ at some point m, one would expect a weakly bound state there with a negative energy, and the model would become unstable. Self-consistency, which simply accounts for the main qualitative effect of the repulsive interaction, serves precisely to prevent this from happening.

Another way to see that weak disorder takes the system away from, and not towards, the critical point, is to compute the ground-state energy perturbatively in weak Gaussian disorder. To the second order in disorder this leads to $\epsilon_0 > 0$ at $\mu = \mu_{\mathrm{c,pure}}$, so with disorder $\mu_{\mathrm{c}} > \mu_{\mathrm{c,pure}}$ at least, in contradiction with Hastings. Also, at infinite disorder one can neglect the kinetic energy (gradient) term and the self-consistent equations decouple at different sites. The problem then becomes $0+1$ dimensional so $\mu_{\mathrm{c}} = +\infty$, again in contradiction to Hastings' claim $\mu_{\mathrm{c}} < \mu_{\mathrm{c,pure}}$. This is in accordance with our assertion that disorder decreases correlations.

Introducing disorder into the large-N model therefore always *decreases* all $J_{i,i}$ at the critical point of the pure system, so the Griffiths inequality implies that correlations decrease too. This does not necessarily mean that there is no transition, as correlations may still exhibit long-range order, only weaker. It does mean, however, that the existence of the transition in the disordered problem is not a logical necessity. In fact, the existence of the superfluid transition would imply that the ground state ϕ_0 became extended, in spite of being an eigenstate of the random (albeit correlated) potential $V(\vec{x}) + \chi(\vec{x})$, as in equation (4). From the point of view of Anderson localization this would be extremely interesting, and could have consequences for other interacting and disordered problems. But, as we argued in the paper [1] and here, it does not seem to be the case in the theory (1) with $D = 2$.

References

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[2] Hastings M B 2001 *J. Phys. A: Math. Gen.* **35** 2519 (preceding Comment)
[3] Griffiths R B 1969 *J. Math. Phys.* **10** 1559
[4] See equation (1) in: Campbell M 2001 *Commun. Math. Phys.* **218** 99